

Bridges: A Journal of Student Research

Issue 6

Article 5

2012

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Kern, Kevin (2012) "Multimessenger Astronomy: Modeling Gravitational and Electromagnetic Radiations from a Stellar Binary System," *Bridges: A Journal of Student Research*: Vol. 6 : Iss. 6 , Article 5.
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Multimessenger Astronomy: Modeling Gravitational and Electromagnetic Radiations from a Stellar Binary System

--Kevin Kern--



Kevin Kern grew up in the Grand Strand Region and graduated from the Scholars Academy in 2009. He came to Coastal Carolina University to study applied physics and also has interests in mathematics and computer science. As an undergraduate, he has completed research projects in each of these areas of interest. In 2010, he was selected as a NASA Ambassador, which involved creating a public outreach program. In 2011, Kevin spent the summer at the University of Southern Florida participating in the Research Experience for Undergraduates program. While at the University of Southern Florida, he performed research in the area of numerical biophysics. Upon graduating from Coastal Carolina University, Kevin hopes to continue graduate studies on the topic of Gravitational Wave Astronomy.

ABSTRACT

Our Sun is one of roughly 100 billion other stars that make up the Milky Way Galaxy. Two-thirds of all stars are paired off, with a gravitational bond between the two stars. Such systems are known as stellar binaries. Although these binaries are very common in the galaxy, there is much

yet to be learned about their formation, evolution, and interactions. The approach taken in this thesis is to produce simulated data representing the expected measurements that an observational astronomer would collect. We attempt to simulate three different stellar binary star systems: an eclipsing binary, a spectroscopic binary, and a gravitational wave emitting binary. In the case of the eclipsing binary, we aim to create a graph of the amount of light received as a function of time. For the spectroscopic binary, we use the fundamental physical principles to measure the velocity of each of the stars with respect to the Earth. Then for the gravitational wave emitting binary, we generate a plot which measures the distortion of spacetime due to the rotation of the stellar binary. Using these generalized functions, a future researcher will be able to develop a statistical analysis program that combines all of the data from the models in an effort to learn more about the characteristics of the stellar binary.

INTRODUCTION

In astronomy, observers are usually separated by a great distance from the system in which they are interested. Consequently, a multitude of clever methods are employed to learn about the universe. While each method can be applied independently, combining the output from multiple observational and analytical methods generates a more detailed picture of a distant system.

In our galaxy, there are over ten billion stars with almost two-thirds of them in multi-star systems. Astronomers categorize these multi-star systems further based on how many stars are contained within the system. A system consisting of two stars is called a binary system. Astronomers use different observational techniques to measure many physical characteristics of the binary star system such as the time it takes for the two stars to orbit one another, their masses, their separation distance, and the orientation of the system with respect to the Earth. With such a large population of interesting star systems and many observational techniques, astronomers stand to vastly increase our knowledge of the universe through studying these systems.

Three methods for studying binary star systems are observing eclipsing events, tracking shifts in stellar spectra, and detecting gravitational radiation. During eclipsing events, one member of the binary passes directly in front of the other, which changes the total light received from the system. For spectroscopic binaries, shifts in the stellar spectra are observed, and astronomers are able to determine a relationship between the shifts and the motion of each star. While it is expected that all binaries emit gravitational radiation, this form of radiation has not been directly detected. It is expected that through the study of gravitational radiation data, a wealth of new information will be gleaned from the system. Numerical models can be constructed for each of these types of binaries that allow researchers full control of the systems.

Analyzing and comparing all of the possible data we could extract from a given stellar binary system requires an immense statistical knowledge that is beyond the scope of an undergraduate thesis. Instead, in this paper I aim to illustrate a numerical approach to generating synthetic data for the three methods outlined above. The goal is to produce a virtual binary system from which analytical methods that explicitly combine information from each method can be developed.

Motivation for Thesis

Astronomers have a very good understanding of the information we can gather from eclipsing binaries and spectroscopic binaries. So much in fact that it is almost universally present in any undergraduate astronomy textbook. Gravitational wave astronomy is a newer specialized field within astronomy that is currently almost completely theoretical. In contrast with eclipsing and spectroscopic information, gravitational wave information is not represented in any undergraduate texts on astronomy. The research cited in the literature review specifically focuses on gravitational waves because the eclipsing and spectroscopic binaries are so well established. Eclipsing and spectroscopic binaries can inform astronomers about key features associated with a system, such as the orbital period, separation, and stellar size. However, with the inclusion of extra information from analyzing gravitational wave data, more can be gleaned about the same system. This thesis is the first step in demonstrating how much information can be learned by combining the traditional analyses of spectroscopic and eclipsing data with yet to-be-gathered gravitational wave data.

Eclipsing Binary Theory

The technique for observing eclipsing binaries is very well established in the field of astronomy. Only certain orbiting binary star systems that can be seen from Earth have a specific orientation that allows them to be eclipsing binary systems. An eclipsing binary must be apparently edge-on so that when we observe an eclipsing binary, we are able to see the two stars pass in front of one another as they orbit. If we imagine a smaller (secondary) star passing in front of a larger (primary) star, we can see that a certain amount of light would be blocked depending on the orientations of the two stars. A visualization of an eclipsing binary is shown below in [Figure 1](#).

Spectroscopic Binary Theory

Like eclipsing binary systems, spectroscopic binary systems have a well established method of making observations. Instead of measuring the total light in the system astronomers measure differences in the light to learn more about the motion of the stars. The light changes are governed by the velocity of the star in the direction of the Earth by a phenomenon known as the Doppler effect. This is the same effect that causes the siren of an ambulance to sound like a higher pitch when the ambulance is moving towards a listener and then a lower pitch when the ambulance moves away from the listener. The physical manifestation of this change in pitch is the change in the wavelength of the sound wave. A higher pitch corresponds to a shorter wavelength and a lower pitch corresponds to a longer wavelength. A similar phenomenon is observed with light when the source is moving at a high velocity. [Figure 2](#) shows that as a star moves towards the Earth in its binary orbit, the wavelength is shortened, which causes the light to seem more blue ([Figure 2 \(a\)](#)). The opposite case is where the star is moving away from the Earth. The wavelength is increased in that case which causes the light to be more red ([Figure 2\(c\)](#)). In astronomy, we call the shifting of the light either blueshifted or redshifted respectively. An important third case is where the star is not moving either from or towards the Earth, such as when the secondary is passing in front of or behind the primary. In this last case, the light is neither blueshifted or redshifted, but rather it stays the same wavelength ([Figure 2\(b\)](#)). By

measuring how much the light has been either blueshifted or redshifted, we can determine the velocity of the star.

Gravitational Wave Theory

Gravitational waves are created when a mass distribution varies with time. Stellar binaries are a great source for gravitational waves because their mass distribution naturally varies with time due to their orbital motion, and also because they are massive enough to generate gravitational waves with sufficient signal strength to be detected. General relativity predicts everything in the universe lives within a 4-dimensional space-time, with three spatial dimensions and one time dimension. A 2-dimensional analog to this 4-dimensional concept is to imagine an immense rubber sheet upon which all objects sit. The deformations caused by these objects in this sheet dictates how gravity influences the motion of other objects. A prediction of general relativity and Einstein's equation is that when a massive object moves in spacetime, it emits a gravitational wave (Baez and Bunn, 2005). A gravitational wave can be thought of as a moving ripple through the “sheet” of spacetime (Hulse and Taylor, 1975). These ripples cause objects to periodically compress and expand. We quantify the expansion or compression by measuring the strain, which is the ratio of the change in size of the object to the unaffected size. If we can understand how to detect gravitational waves, we can learn much more information about our galaxy.

Just like there is a spectrum of electromagnetic waves, there exists a spectrum of different frequencies of gravitational waves with each one carrying a different type of information (Centrella, 2003). There exists a special kind of stellar binary whose components are just black holes. This black hole binary system is hard to get information about using electromagnetic radiation because these systems do not emit light very well. Gravitational waves are not hindered by this limit and can be measured even when two black holes are spiraling in towards one another (Yunes et. al., 2008).

Gravitational Wave Detectors

Gravitational wave detection is small part of astronomy currently because it is an emerging field. Relativity predicts gravitational waves radiate away energy and angular momentum from a binary system. Hulse and Taylor (1975) were the first to show indirect evidence of gravitational waves by measuring the period of a pulsar star system. A pulsar star is a star that emits a “beam” of electromagnetic radiation that can be measured using instruments at observatories. By monitoring how often these radiation pulses were observed, Hulse and Taylor showed that the binary star system was increasing in orbital speed, indicating a loss of energy and angular momentum. In the years since their indirect measurement of gravitational waves, it has been the goal of many researchers to directly detect gravitational waves.

Detecting gravitational waves directly is difficult because they cause very small measurable effects. Gravitational waves are 40 orders of magnitude weaker than electromagnetic forces. Because the waves are so weak, detectors must be both accurate at measuring small changes in the signal and able to see through interference to find such a weak signal (Lewis, 1995). A common approach for searching for gravitational waves is to use a laser interferometer gravitational wave detector. This type of detector operates by shooting a laser down two

perpendicular tunnels towards a mirror and measuring the time it takes for the round trip. Since gravitational waves distort space, one tunnel will be longer than the other and the light will take a longer time (Raab, 2008). Other types of detectors have been proposed that use different architecture and techniques to measure higher frequency gravitational waves which have different characteristics than lower frequency waves (Giampieri & Polnarev, 1997; Li & Baker, 2007). Bar detectors are a type of detector that can measure higher frequency gravitational waves, but this type of detector only operates over a small range of frequencies. An interferometer device, on the other hand, is a broad-band detector that is capable of measuring gravitational waves from a large range of frequencies. The technical details of how these high frequency detectors operate is beyond the scope of this thesis, but it is important to realize there are more detector types than just the interferometers.

Another clever way researchers increase the odds of detecting gravitational waves is to focus on data from binaries that are predicted to yield high amounts of gravitational wave information for the type of detector being used. This seemingly simple idea is hard to put into practice because gravitational wave detectors are not able to be focused into one area of the sky. One of the most prevalent groups in gravitational wave research today is The LIGO Scientific Collaboration and the Virgo Collaboration. In a 2010 report, the collaboration demonstrates their method of data analysis of wave data from sources that are relatively close to the Earth. According to the report, the collaboration was unable to detect gravitational waves because the detectors used were not precise enough. In effort to observe the gravitational spectrum directly, NASA and the European Space Agency (ESA) proposed a mission to make an enormous light interferometer in space called the Laser Interferometer Space Antenna (LISA) that would specifically target low frequency gravitational waves and serve as a complimentary gravitational wave detector to the LIGO/Virgo collaboration. [Figure 3](#) displays an artist's rendition of LISA. In recent years, NASA has withdrawn from this project leaving its launch status questionable. The idea behind this type of interferometer is the same as the ground-based one, monitor a laser beam between two photodetectors and measure phase shifts in the light. Benefits of this detector being in space include less noise from seismic activity and free vacuum sources. Difficulties arise in the fact that the distance between the detectors would not always be held constant because of fundamental orbiting principles. This type of detector could possibly open up a new range of low frequency gravitational waves for direct observation (Armstrong et. al., 1999). The LISA detector was in development for so long that multiple follow-up missions were proposed to increase sensitivity in the devices and upgrade the technology to increase the chance of direct detection (Bender, 2004).

Gravitational Wave Simulations

Many proof-of-concept simulations were made for the LISA detector, including simulations which show dry-runs of the expected data. Benacquista et al. (2004) demonstrate a simulation technique of building an entirely new population of stars and calculating what the measured data should be for the LISA detector. The goal of our numerical simulation is to produce synthetic data which can be used to develop statistical analysis routines for extracting physical characteristics associated with the binary system. By performing an experiment where we control the source of gravitational radiation, we can verify our analysis routines. This is in contrast to observational astronomy where we have no control over the emitting source, and therefore have

difficulties verifying our analysis routines. Blaes (1998) provides details about the possible improvements to our understanding of mass transfer in binary systems from gravitational wave data gathered from the LISA detector. Inside our own galaxy there are about ten billion gravitational wave emitting binary star systems. Through advanced statistics researchers are able to identify and subtract noise from simulated data to get a better view of the actual data instead of noisy data from all the sources in the galaxy (Rubbo, 2004). Puchades and Sáes (2006) are doing the exact opposite; they are taking the individual expected data from a number of binary star systems and superimposing all of the data to one curve. This analysis shows what a plausible data curve for the LISA station could look like if there were no other interference and only pure data from many binary systems. Other researchers are looking at current algorithms for calculating the wave data from binaries and trying to find more efficient and less costly methods of obtaining results that are at least as accurate as before. Baker et al. (2006) generate a gravitational wave model from a binary black hole system that is accurate over a much longer timespan than a previously approximated model. This kind of improvement helps determine more information about a given system by allowing the researchers a much larger data set. A better and more efficient algorithm is a way to save memory, so the computer can process a larger data set and show researchers more possible data combinations (Cannon et. al., 2011).

Methods

The scope of this project was to illustrate a method of constructing a numerical simulation for data received from stellar binary star systems. This model was constructed using the Mathworks Matlab coding environment. This environment was chosen because of its ability to handle large amounts of data using an efficient computing algorithm. The code focused on three types of simulated data: light curves from eclipsing data, motion information from spectroscopic data, and gravitational-wave time series data. To simplify the calculations within the code a few assumptions were made regarding the binary system in each case. One assumption applied to all cases was that these stars could be modeled by spheres of uniform density.

For the eclipsing light curve data other assumptions of motion were made. The first assumption was that the stars orbit with constant angular speed. The second assumption was that the stars were moving mostly towards and away from the Earth; a motion known as edge-on rotation. We always know some information about the star system from observational methods and in the case of the eclipsing binary we know the angular velocity of the secondary star (the smaller of the two stars), how long it takes the secondary to pass in front of the primary, and the total amount of light given from each star (luminosity). By calculating the luminosity of the whole system we know a baseline measurement of how bright the binary system is. Then, as the secondary progressed in front of the primary, we expected the total luminosity to decrease because the secondary would block out light from the primary. This experiment was continued to the case where the secondary passed behind the primary to get a light curve for the whole rotation of the binary star system. It is important to note that the simplifications made in this type of simulation only make sense because we know the theory behind the physical process. In general, all simulations should try to make as few simplifications as possible.

For the case of the spectroscopic binary, we know the masses of both stars, the frequency of the system, and how often we observe the system. The observation variable was included to more

closely simulate real data collection. Data cannot always be collected at even intervals over the course of a real observational experiment because of problems caused by the weather or other unforeseen technological complications. We do not make the edge-on motion assumption for this calculation, and as a result the calculation for the velocity curve was much more general and could apply to more binary systems. Upon completion, the program generates a plot of all the data in the form of a velocity curve.

For the gravitational wave code we know the amplitude of the gravitational wave signal and the location and orientation of the binary system. Using this information we are able to generate an ideal gravitational wave output signal from the binary system. Then we use this ideal signal and pass it through a filter that simulates how the detector alters the signal. After applying this response function to the incoming signal we plot the expected gravitational wave detector output.

Eclipsing Binary Findings

The validation for this code comes from our understanding of the eclipsing binary star system. We know if the stars start out side-by-side, the total light from the system should be just the sum of the light from each star. When the secondary begins to transition in front of the primary, we know the light should decrease by a large amount as more and more of the primary is blocked from view. When the secondary is fully in front of the primary, we know the amount of light becomes constant at a minimum value. Later in the orbit, the secondary begins to transition outside of the primary again and the light increases back to its original value. When the secondary transitions behind the primary, the total light decreases yet again because the primary is blocking the light from the secondary. The light curve decreases until the secondary is fully behind the primary and becomes a constant again. The final part of this motion is when the secondary begins to transition back to its original position and, as before, the light curve goes back to its total value for both stars.

An example of the output from an eclipsing binary light curve simulation is shown in [Figure 4](#). Notice the characteristics described above in the validation section are all present for this data. At the beginning of the curve, the light is at a maximum, implying the two stars start out next to each other. There is a very sharp drop to a minimum value showing the secondary star passing in front of the primary. The maximum light is attained again while the secondary is next to the primary again. A much smaller dip is shown as the secondary passes behind the primary.

Spectroscopic Binary Findings

The validation for this code consists of a few important comparisons between different simulations. The first case we check is the case where both stars in the binary have the same mass. We know that if two stars have the same mass, they will both be contributing equally to the total energy of the system. We know energy is dependent on both the mass of the star and the velocity of the star squared. If both stars have the same mass and are contributing equal amounts of energy, they must have the same velocity. The output of the spectroscopic binary code shows the velocity with respect to the Earth, so a negative velocity would be going towards the Earth and a positive velocity would be going away. If we look at the amplitude of the velocities of the two velocity curves of equal mass, we expect them both to be equal. If the masses of the two

stars in the binary are different, we expect them to rotate at different speeds. The total amount of rotational energy in the system must remain constant, no matter what mass configurations make up the system. We know by Newtonian mechanics that the primary must move slower because it is more massive and the secondary would move faster. If we take an extreme case of this relationship and make the primary much larger (about 100 times the mass of the secondary), we expect to see the primary's velocity to be small and the secondary velocity to be large.

Shown below are three different simulations that illustrate the verification of the code. [Figure 5](#) is the equal mass binary and we can see that the curves amplitudes are identical. [Figure 6](#) is the case of a slightly larger primary where the primary is ten times the mass of the secondary. We see in this case that the primary has an amplitude of about 5 km/s where the secondary has an amplitude of 40 km/s. This definitely is consistent with the verification that as the mass of the primary increases, the velocity of the primary decreases and the velocity of the secondary increases. [Figure 7](#) is the extreme mass ratio case where the primary mass is 100 times the mass of the secondary. It can be seen that the amplitude of the primary is very small and almost non-existent on the velocity curve, but the amplitude of the secondary is over 80 km/s. This is consistent with the idea that as the mass ratio continues to change so does the velocity, even at extreme mass ratios.

Gravitational Wave Findings

This simulation is focuses on binaries that have an orbital period of around one day. Because the period is so low, the orbital velocity is nearly constant. Low frequency gravitational wave emission of this type is usually a sine curve that depends on how the velocity of the binary system changes. In the case where the orbital velocity does not change, the emission follows a simple sine curve that does not decay over time.

Gravitational wave data is shown in time series plots that show the strain versus time. [Figure 8](#) shows the gravitational wave time series graph over a period of 2000 seconds of a binary whose stars are close to a solar mass. The numerical simulation makes a constant orbital velocity assumption for simplification. The general curve of this strain versus time graph looks like a sine curve as is expected with the constant orbital velocity approximation over a small time period. Fourier analysis must be performed to generate useable information from a time series graph of varying velocity, but this type of analysis is beyond the scope of this thesis.

Conclusion

The central outcome of this thesis is the production of three interrelated codes that produce synthetic data for a single, hypothetical binary star system. While methods for studying eclipsing and spectroscopic binaries are well established, this thesis incorporates gravitational wave data through the use of simulations for the first time. This is a key step in the progression of gravitational wave astronomy in that it demonstrates the potential wealth of knowledge that can be gained through observations of gravitational radiation. To date, gravitational waves have not been directly observed. However, multiple governments have, or will be investing, billions of dollars in gravitational wave research. Prior to such a commitment it is important to demonstrate the potential return on the investment. This project is the first in a line of proof-of-concept

projects to estimate what kind of information scientists can expect to receive from a new observatory. The scientists will know an approximation of how much information they stand to gain from building more powerful gravitational wave detectors. The synthetic data produced in this thesis will be used in a future project dedicated to statistical analyses that specifically address how the data are combined and to what extent new information can be gathered. This approximation can then be conveyed to engineers who, in turn, estimate a cost of the new detector. The next step in measuring gravitational waves can only be completed with such a cost-benefit analysis to prove its financial stability.

FIGURES

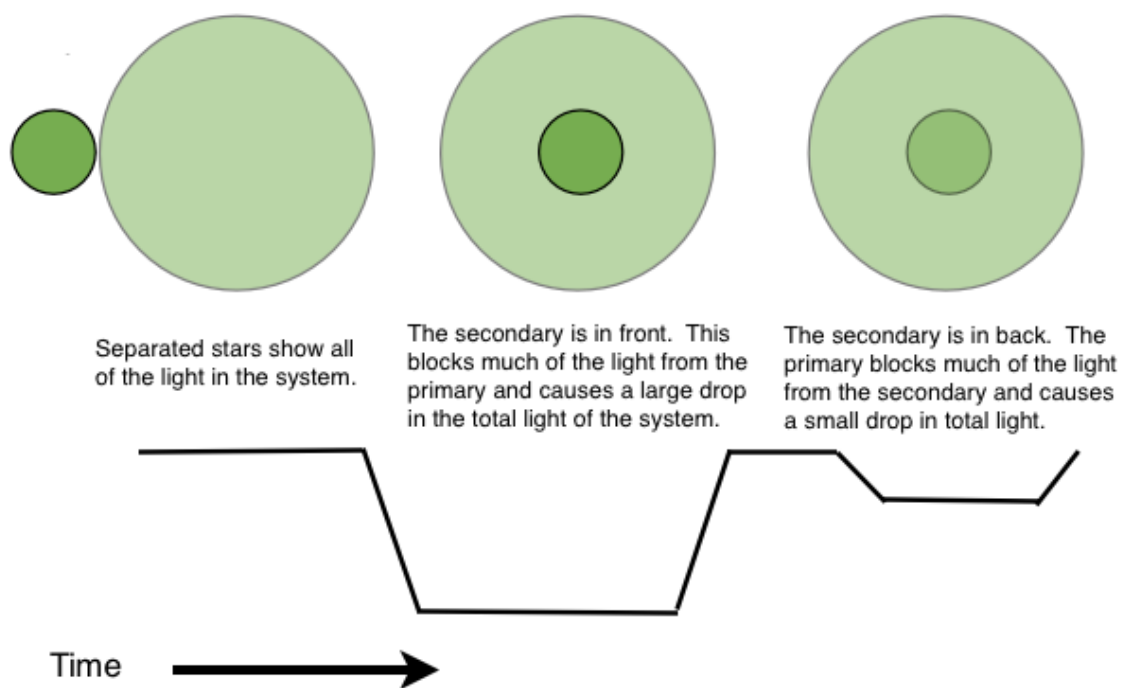


Figure 1: Visualization of an Eclipsing Binary

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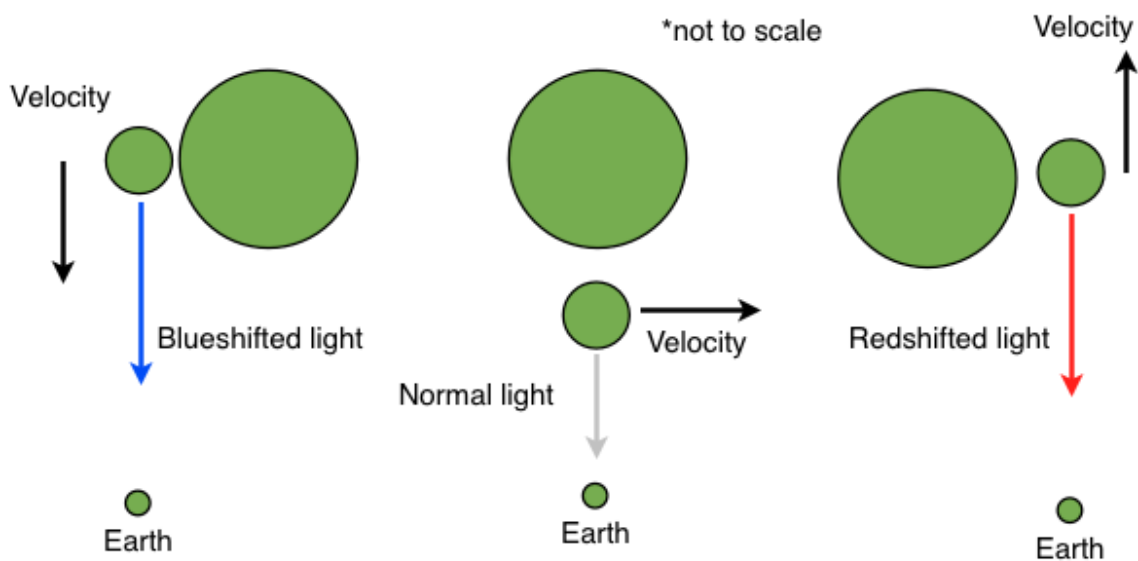


Figure 2: Blueshifted light (a), Normal light (b), and Redshifted light (c)

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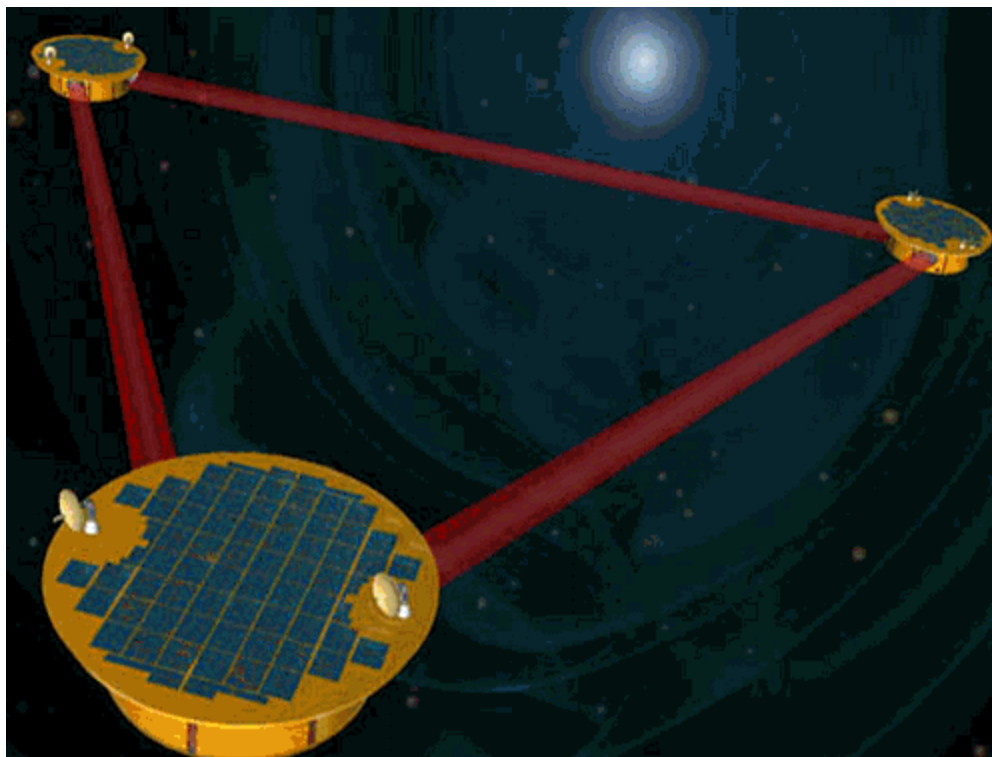


Figure 3: A NASA artist's rendition of the Laser Interferometer Space Antenna (LISA)

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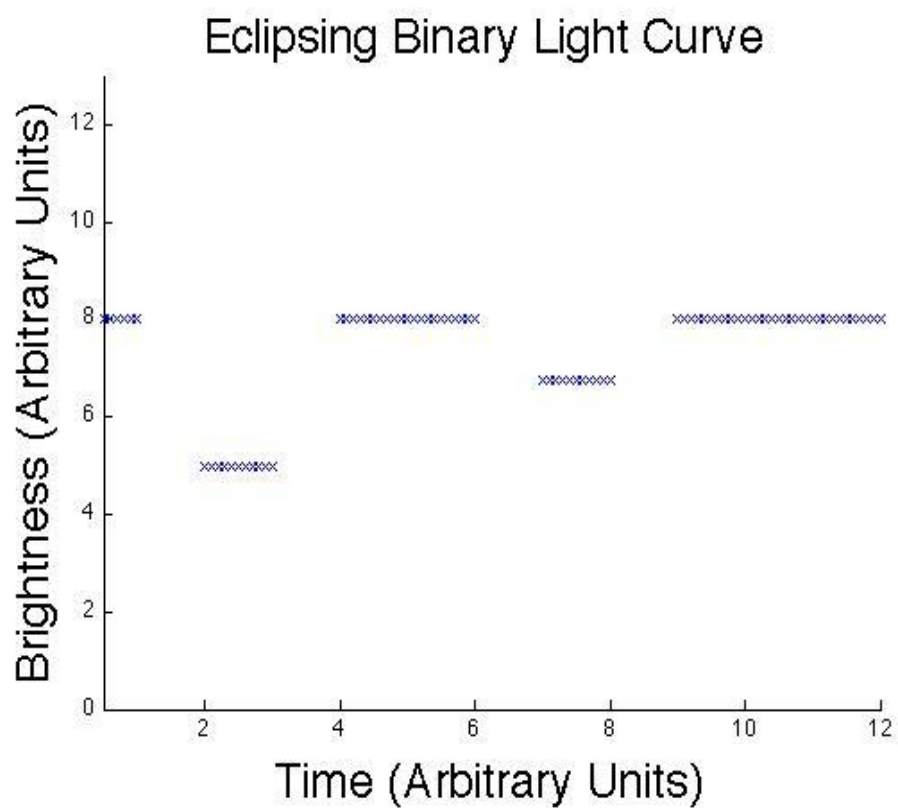


Figure 4: A light curve from an arbitrary stellar binary system.

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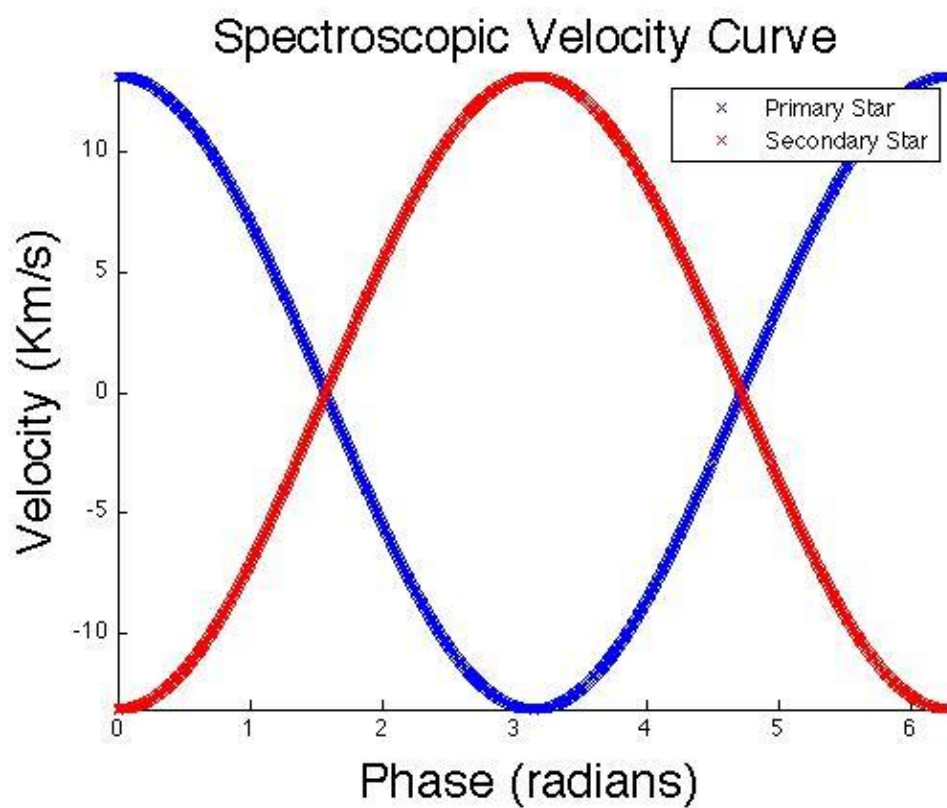


Figure 5: An example velocity curve where the primary mass is equal to the secondary mass.

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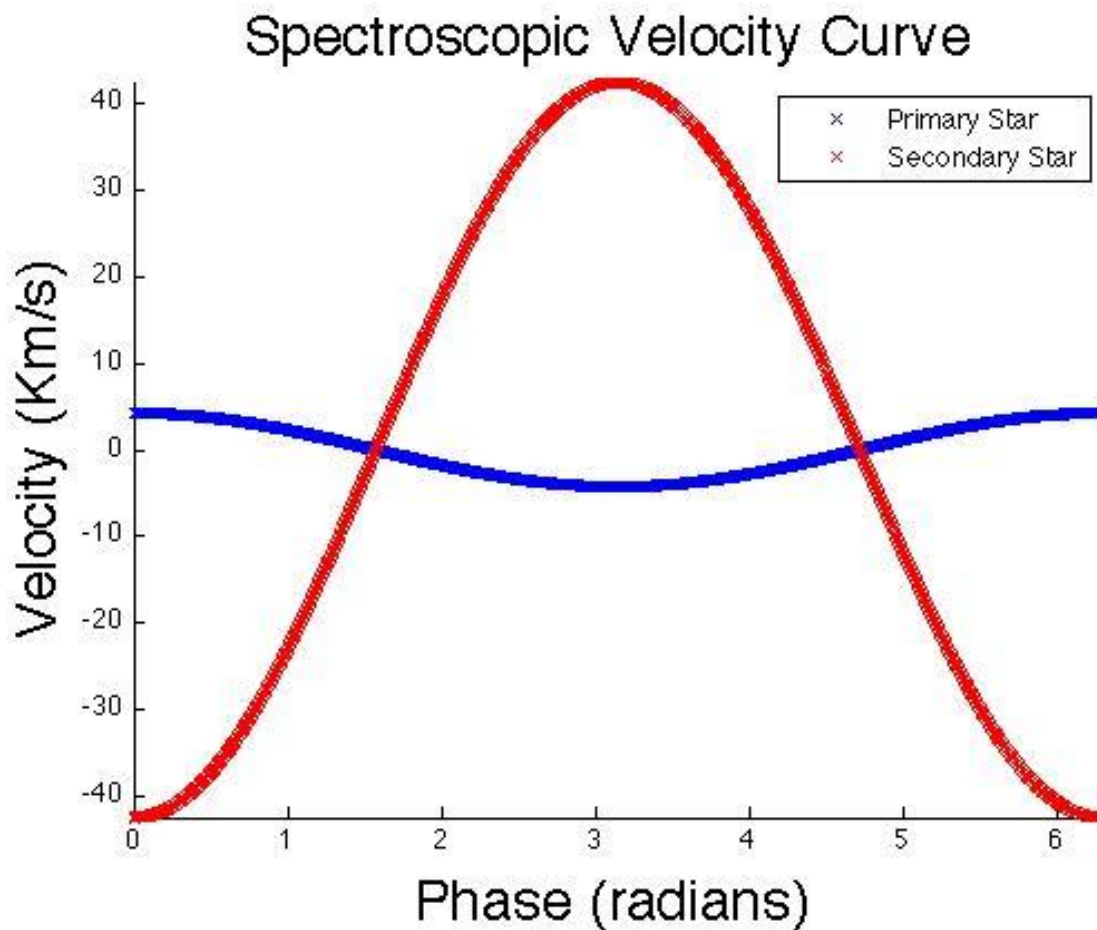


Figure 6: A spectroscopic velocity curve with a mass ratio of 10:1. Note that there is a large difference in the amplitude of the velocities.

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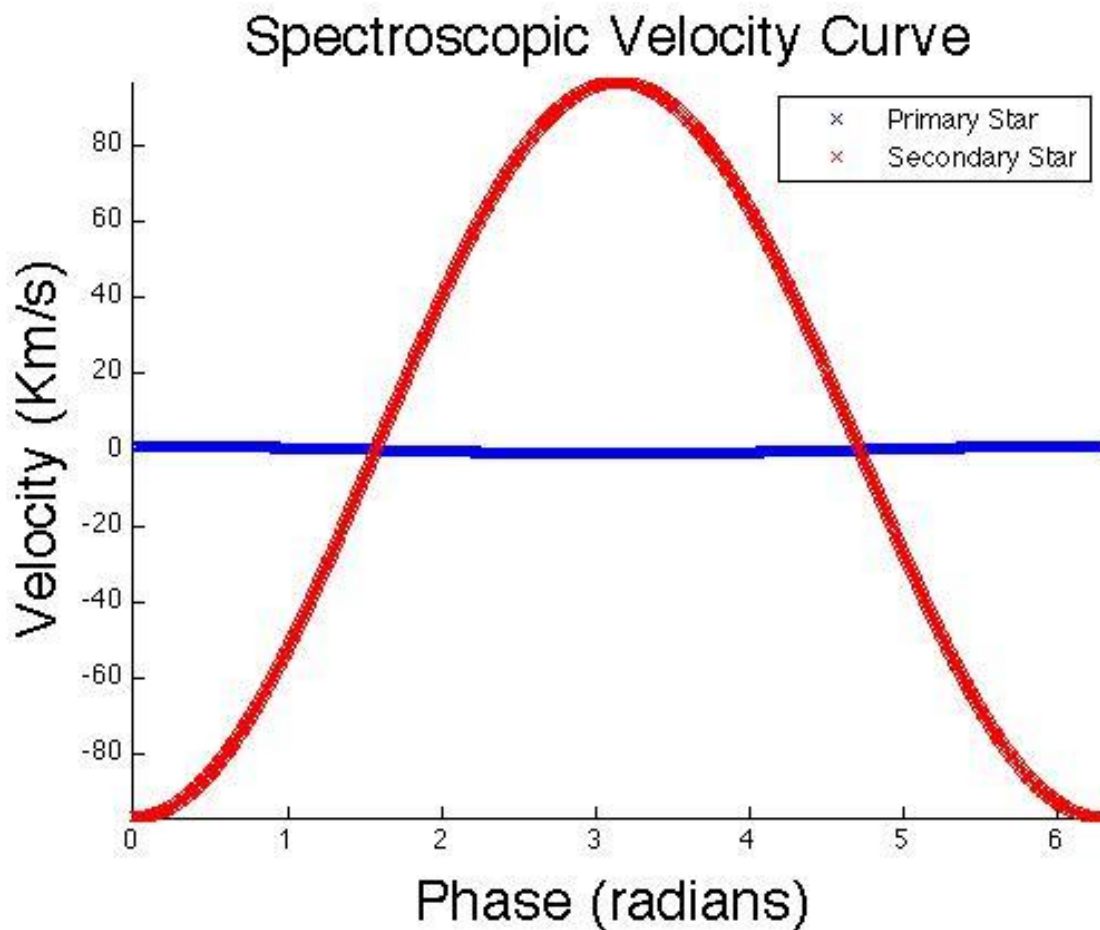


Figure 7: A spectroscopic velocity curve with a mass ratio of 100:1. Note that there is an even larger difference in the amplitudes of the velocities than in the curve illustrated in Figure 6.

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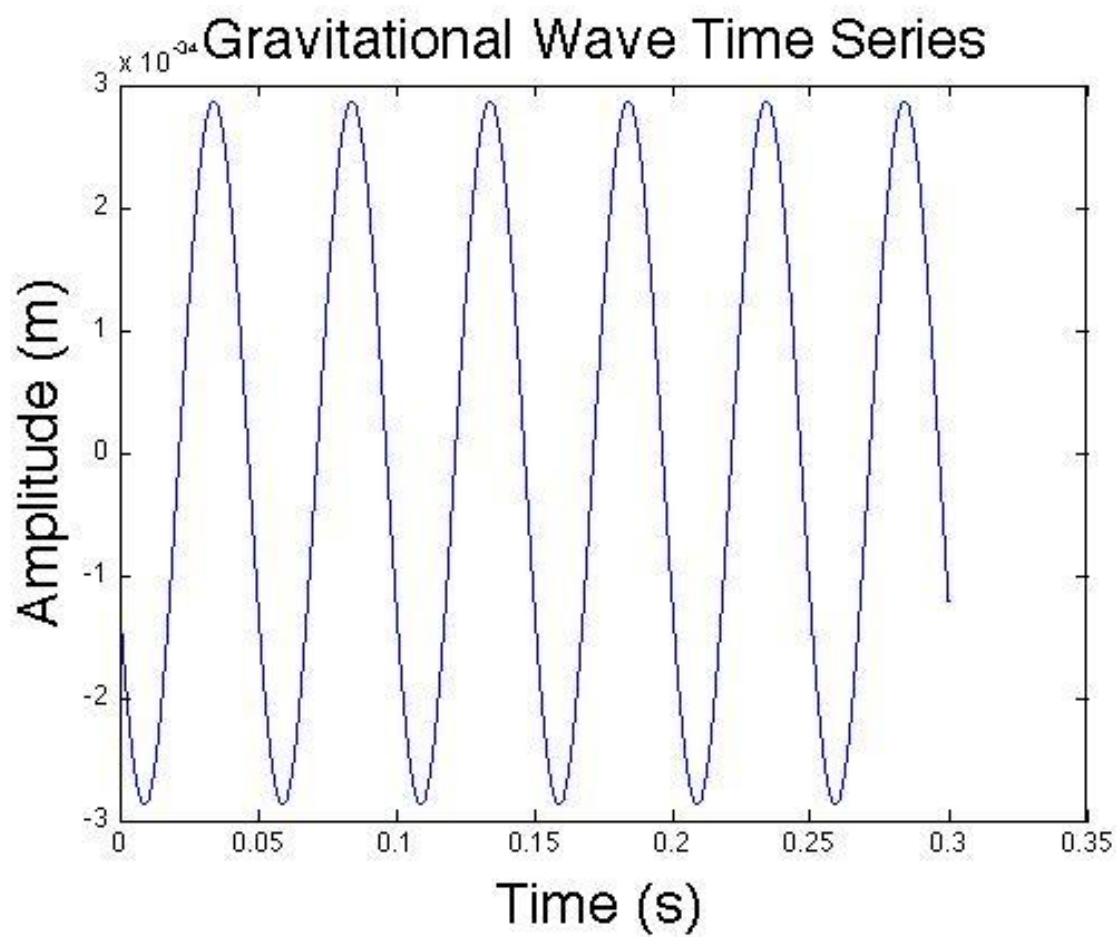


Figure 8: Strain vs. time curve for gravitational waves.

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